Non-Gaussian Normal Diffusion in a Fluctuating Corrugated Channel

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International Conference on

Transport Phenomena in Complex Environments

September 3-6, 2019 – Erice (Sicily), Italy

Outline

- non-Gaussian normal diffusion: experimental evidence

- from Laplace (exponential) to Gaussian distribution, same diffusion constant: the diffusing diffusivity argument

- a microscopic model: Brownian diffusion in a fluctuating corrugated channels

- general remarks

Non-Gaussian Normal Diffusion

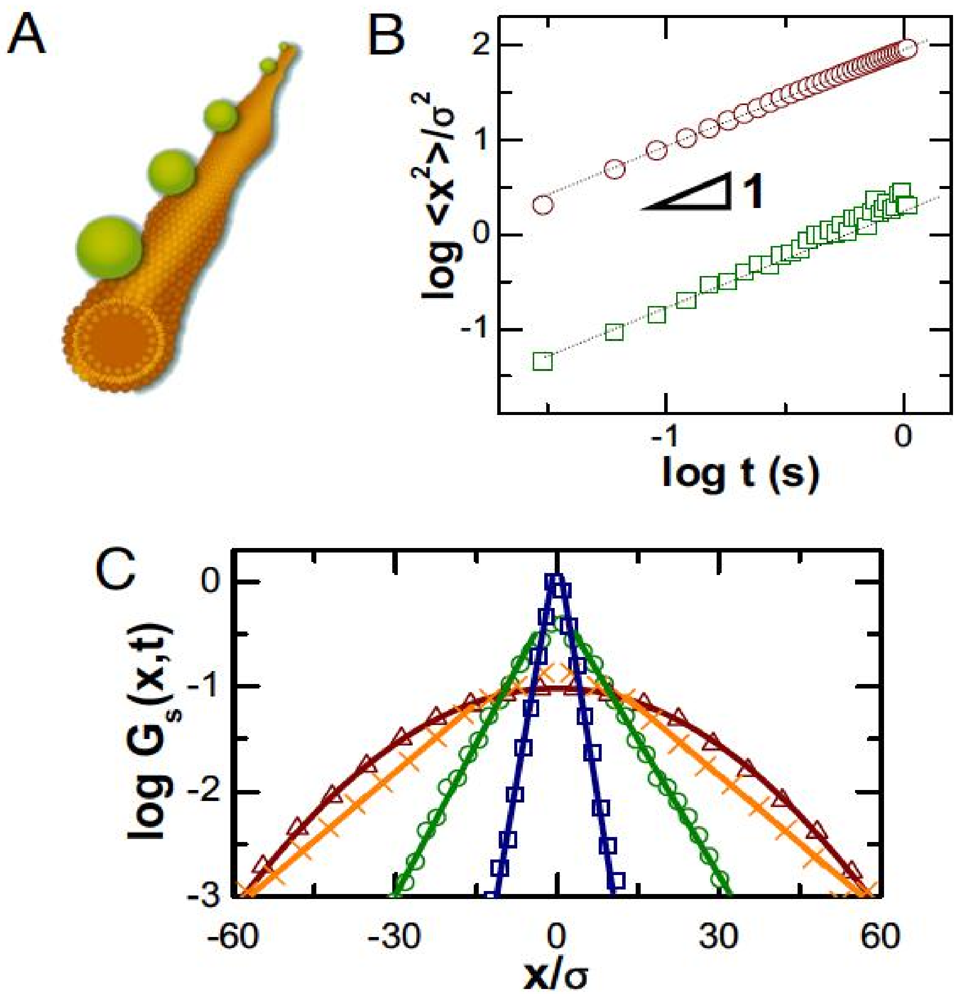
key features:

- normal diffusion $\langle x^2 \rangle = 2Dt$, same D in both regimes

- pdf scaling $P(x/\sqrt{t})$
  - Laplace (short t)
  - Gaussian (large t)

key ingredient:

Slow varying environment



PNAS, 106, 15160 (2009)

Anomalous yet Brownian

Bo Wang[a], Stephen M. Anthony[b], Sung Chul Bae[a], and Steve Granick[a,b,c,d,1]

Colloidal beads in entangled actin Suspensions

Wang, Granick, PNAS 106 15160 (2009)

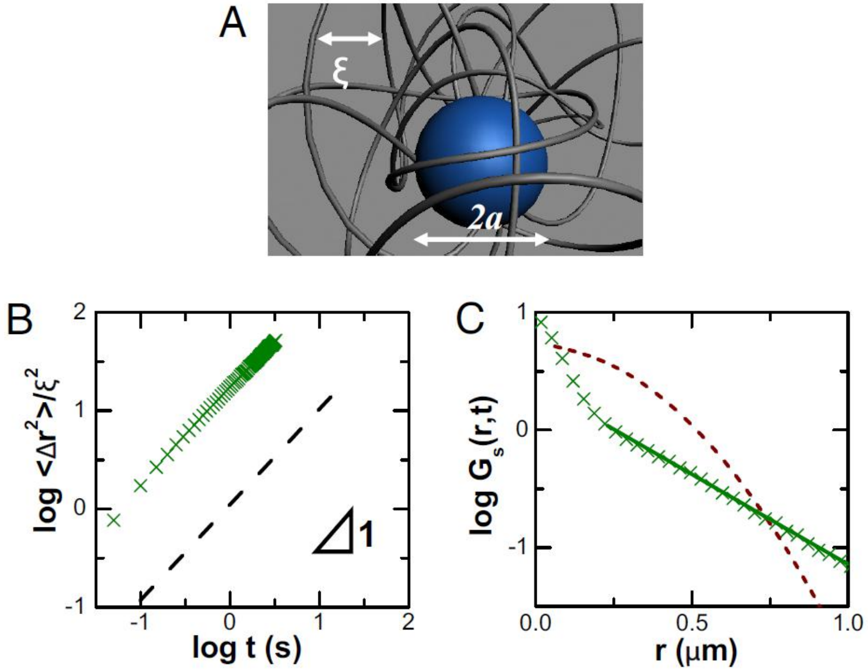
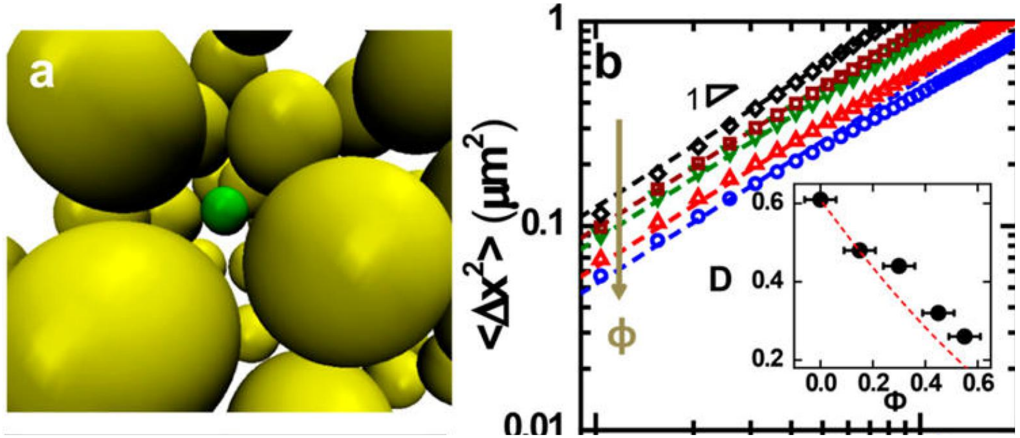
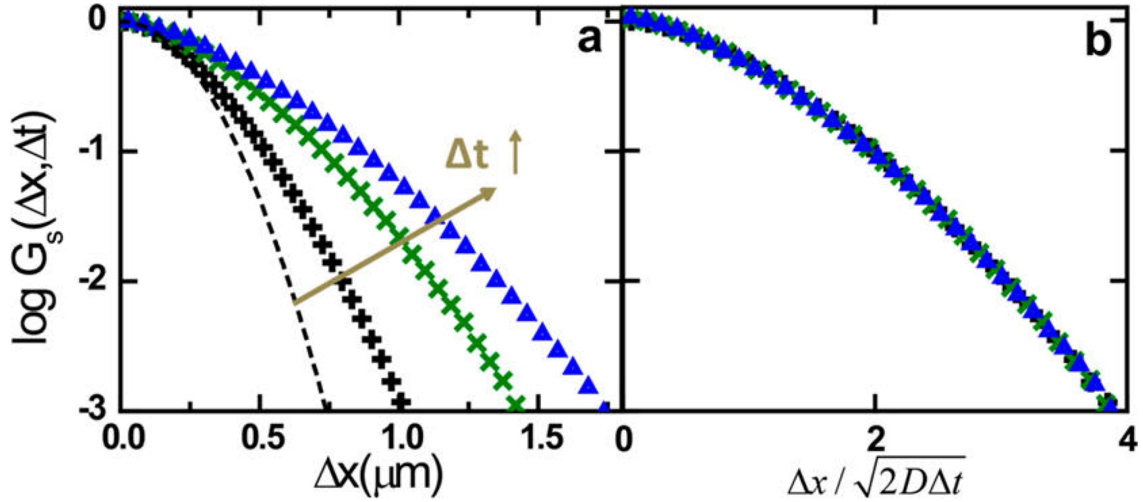


Fig. 3. The second system: Nanospheres diffusing in entangled actin. (*A*) Schematic representation of particles diffusing in entangled actin networks. The mesh size (average spacing between filaments) in nanometers can be estimated as $\xi = 300/\sqrt{c}$, where *c* is actin concentration in milligrams/milliliter. Their concentration is semidilute. The average particle–particle separation is ≈ 10 μm and their radius is a = 25–250 nm. (*B*) Mean-square displacement (MSD) normalized by mesh size squared, plotted against time *t* on a log–log scale for particles in entangled F-actin at conditions of a = 50 nm, ξ = 300 nm, showing a slope of unity. (*C*) Corresponding displacement probability distributions $G_s(r, t)$ plotted logarithmically against linear displacement for delay time of 0.1 s. Here, $G_s(r, t)$ can be fitted with a combination of a Gaussian at small displacement and exponential at large displacement (solid line). In *B*, the dashed line is MSD constructed according to the central Gaussian part in the displacement distribution. In *C*, the dashed line shows a Gaussian distribution with the same diffusion coefficient as for *B*.

Hard-Sphere Colloidal Suspensions

Wang, Granick, ACS Nano ***8***, 3331 (2014)





more (earlier) observations

nature COMMUNICATIONS

ARTICLE

Received 2 Jan 2016 | Accepted 20 Apr 2016 | Published 26 May 2016 DOI: 10.1038/ncomms11701 OPEN

Dynamic heterogeneity and non-Gaussian statistics for acetylcholine receptors on live cell membrane

W. He[1], H. Song[2], Y. Su[2], L. Geng[3], B.J. Ackerson[4], H.B. Peng[3] & P. Tong[2]

Three-Dimensional Direct Imaging of Structural Relaxation Near the Colloidal Glass Transition

Eric R. Weeks,[1*] J. C. Crocker,[2] Andrew C. Levitt,[2] Andrew Schofield,[3] D. A. Weitz[1]

www.sciencemag.org SCIENCE VOL 287 28 JANUARY 2000

PHYSICAL REVIEW E **93**, 032144 (2016)

Non-Gaussian normal diffusion induced by delocalization

Jianjin Wang,[1] Yong Zhang,[1] and Hong Zhao[1,2,*]

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(Received 19 October 2015; revised manuscript received 1 February 2016; published 28 March 2016)

PHYSICAL REVIEW E **97**, 042122 (2018)

Non-Gaussian diffusion in static disordered media

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(Received 2 December 2017; revised manuscript received 15 March 2018; published 16 April 2018)

Slow environmental relaxation is common in soft matter, as exemplified

Diffusing Diffusivity

$$\dot{\boldsymbol{x}} = \sqrt{|D(t)|}\boldsymbol{\xi}(\mathrm{t}) \qquad \text{(1D)}$$

$$\dot{D} = -D/\tau + \sqrt{D_0}\ \eta(\mathrm{t})$$

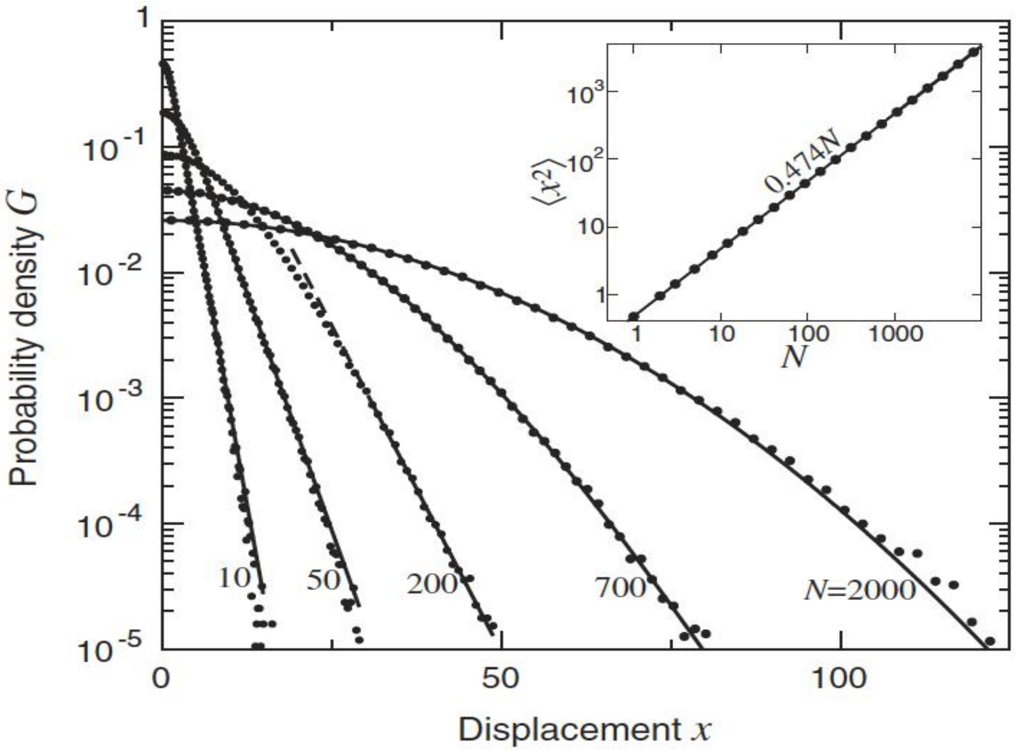
where $p(D) = (1/D_0)\exp(-D/D_0)$

- t << τ: Laplace distribution

$$p(x/\sqrt{t}) = (1/\sqrt{D_0 t})\exp[-x/\sqrt{D_0 t}]$$

- t >> τ: Gaussian distribution

$$p(x/\sqrt{t}) = (1/\sqrt{4\pi D_0})\exp[-x^2/4D_0\mathrm{t}]$$



normal diffusion for any t

$$\langle x^2\rangle = 2D_0 t$$

✓ Unable to predict the Short time Gaussian regime
✓ microscopic mechanism is not clear

Chubynsky & Slater, PRL ***113***, 098302 (2014)

Fluctuating Sinusoidal Channel

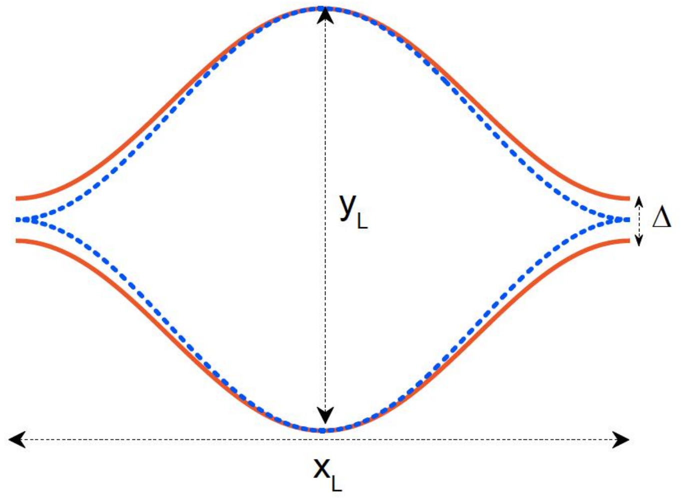
$$\dot{r} = \sqrt{D_0}\,\xi(t)$$

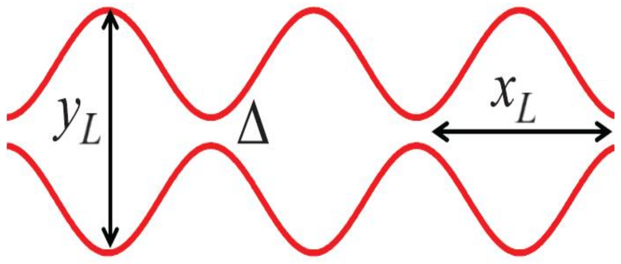
channel boundaries:

$$\omega(x,t) = \frac{y_L}{2}\left[\varepsilon^2 + (1-\varepsilon^2)\sin^2\left(\frac{\pi x}{x_L}\right)\right]$$

pore size: $\Delta = y_L \varepsilon^2\,(t)$

$$\boxed{\dot{\varepsilon} = -\frac{\varepsilon - \varepsilon_0}{\tau} + \sqrt{\frac{D_\varepsilon}{\tau^2}}\,\eta(t)}$$



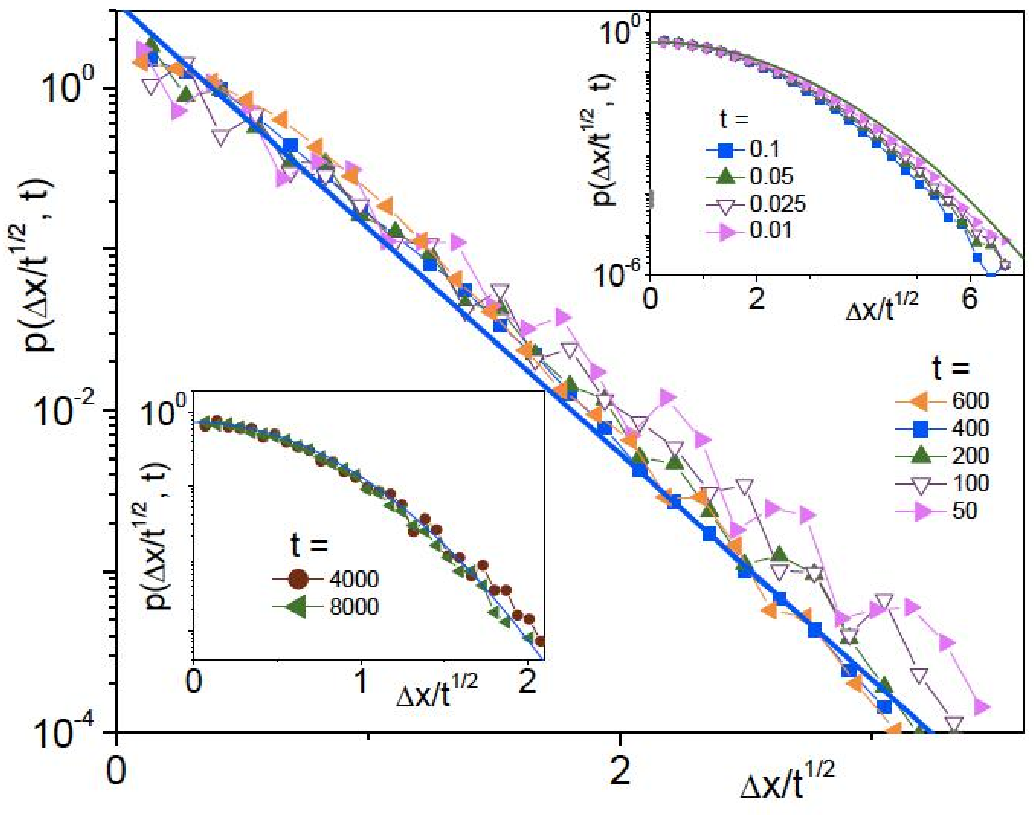


approximations:

- ✓ pointlike particle
- ✓ No hydrodynamic corrections
- ✓ Reflecting boundary conditions

Phys. Rev. Research (accepted)

Main Results (ε_0=0)



- rescaled P(x/√t) are:
  - Gaussian at short t
  - Laplace (exponential) intermediate t
  - Gaussian large t

$$\boxed{P\left(\frac{\Delta x}{\sqrt{t}}\right) = \begin{cases} (4\pi B)^{-1/2} \exp(-\Delta x^2 / 4Bt) \\ (2\alpha)^{-1} \exp(-\Delta x / \alpha\sqrt{t}) \end{cases}}$$

Three time scales

normal diffusion for all t

three time scales

- intracell diffusion time: τ_L

$$\tau_L = \min\{\frac{x_L^2}{8D_0}, \frac{y_L^2}{8D_0}\}$$

$\boxed{B = D_0}$

- mean first passage time (MFPT): τ_0

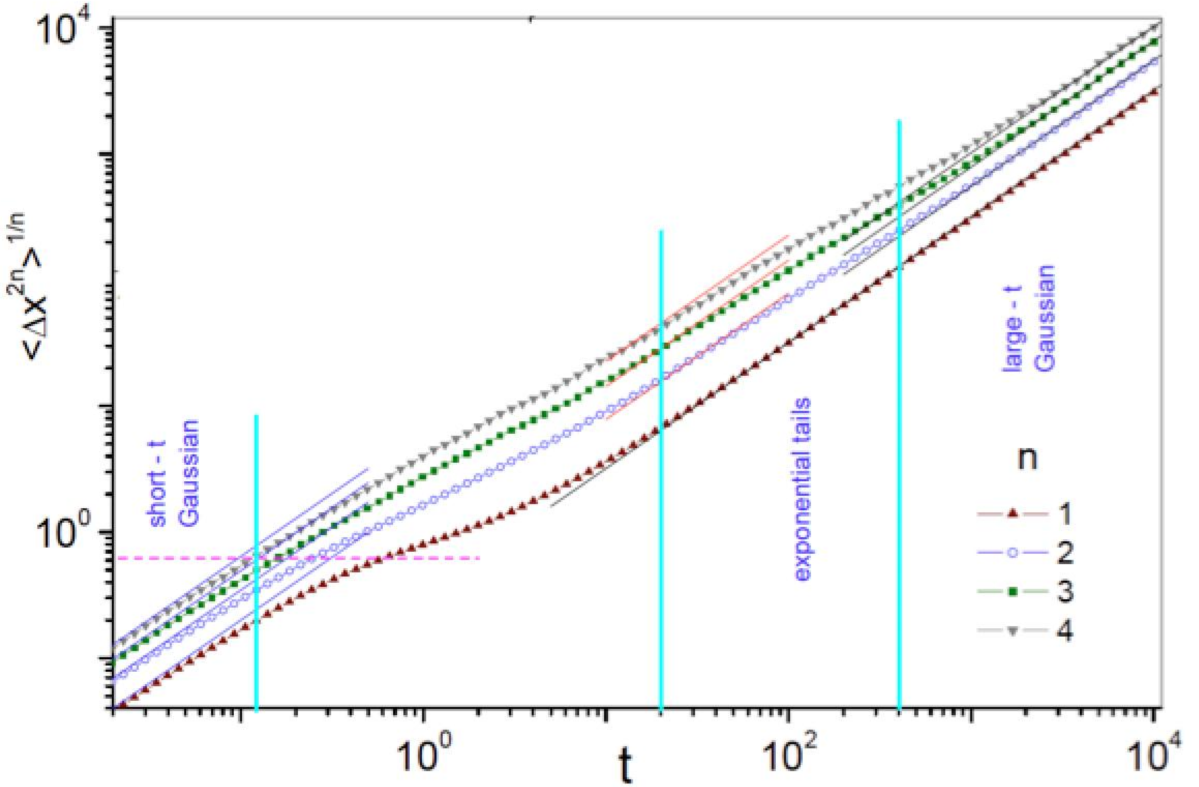
$$\tau_0 = \frac{x_L^2}{8D_0}\frac{1}{\langle|\varepsilon|\rangle}$$

$$\langle|\varepsilon|\rangle = \sqrt{\frac{2D_\varepsilon}{\pi\tau}}$$

- fluctuation correlation time: τ

τ

$$\boxed{B = D = \frac{x_L^2}{4\tau_0}}$$



τ_L $2\tau_0$ 2τ

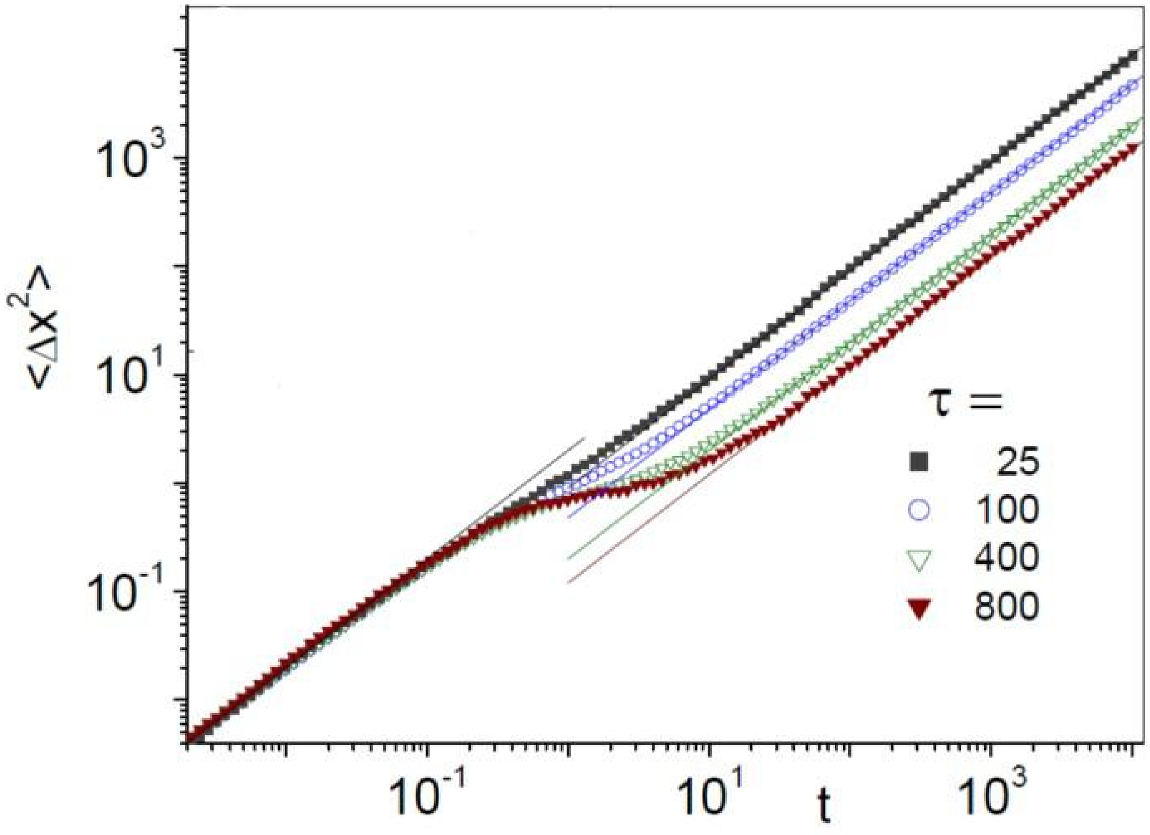
Fluctuating time scale

- all curves collapse on linear branch with diffusion $B=D_0$ for short time

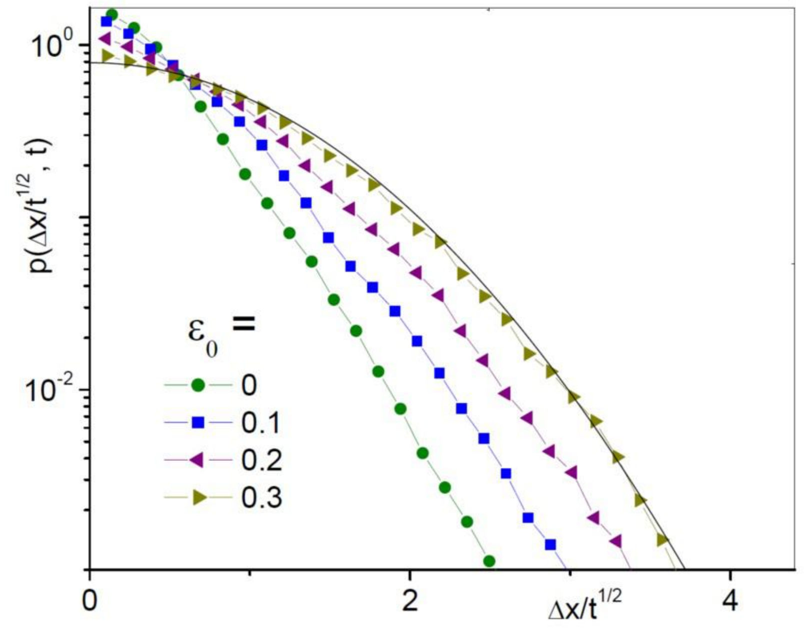
- Linear fits with large time B

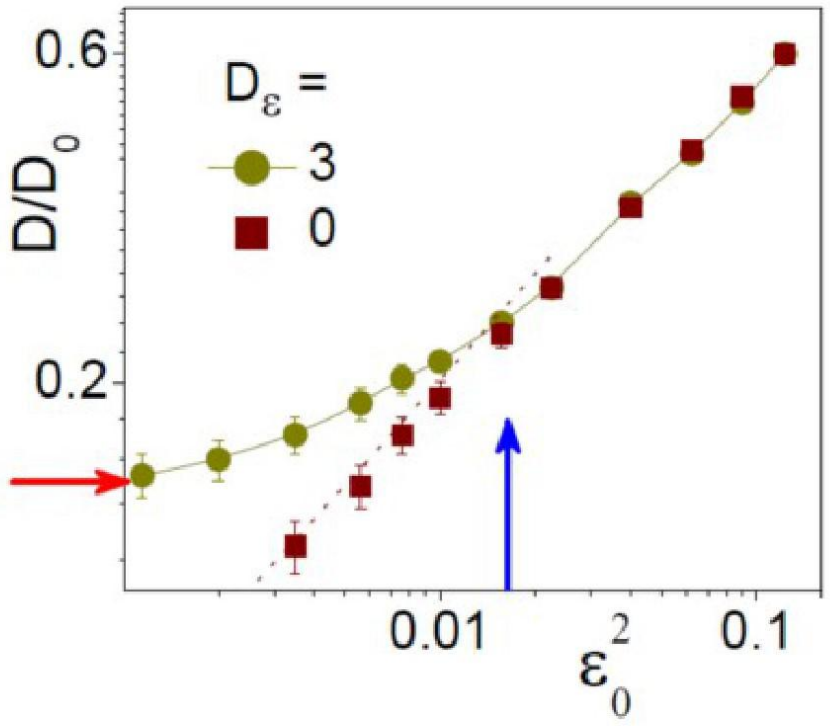
- Lowing τ, the slopes coincide

- Plateau in the range $\tau_L << t << \tau_0$
  - particle fill up the channel well



Pore Size





- ➢ widening the pores, exponential to Gaussian

- ➢ Non-Gaussian normal diffusion only occurs when the channel fluctuations are strong enough to actually open/closing.

Conclusions

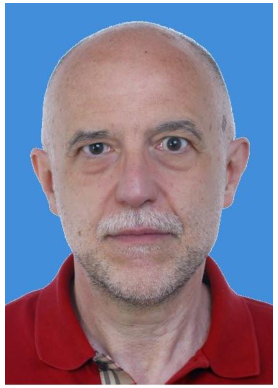
- simple microscopic model exhibiting non-Gaussian normal diffusion

- all main features of the phenomenon $\dot{\varepsilon} = -(\varepsilon - \varepsilon_0)/\tau + \sqrt{D_\varepsilon/\tau^2}\,\eta(t)$

- pore opening-closing play a key role

Acknowledgement

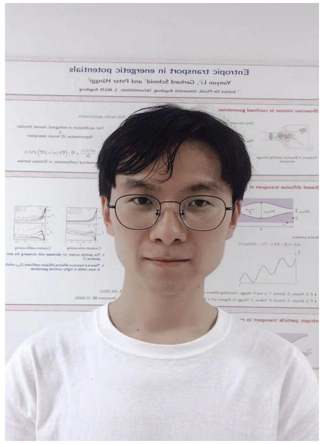
Collaborators:



Prof. Fabio Marchesoni

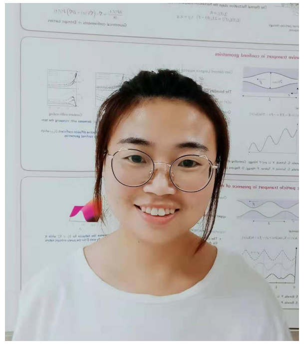
Prof. Pulak Ghosh

PhD

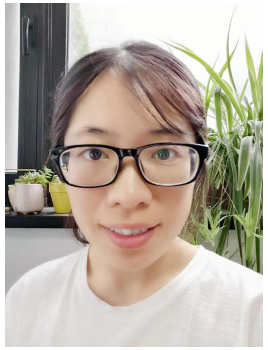


Qingqing Yin

Graduate students



Lihua Li



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国家自然科学基金委员会
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